REFLECTING MICROSCOPES

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ABSTRACT. The history of reflecting microscopes is reviewed, and the leading Schwarzschild design formulae quoted. Two reflecting objectives are described, and photomicrographs taken with one of these are reproduced.

§1. HISTORICAL INTRODUCTION

REFLECTING microscopes are almost as old as reflecting telescopes, for it was early realized that if the light be sent backwards through a telescope objective, it becomes a microscope objective, albeit of inconveniently great focal length. Newton made a reflecting microscope objective consisting of an ellipsoidal mirror and a diagonal flat; a two-mirror objective more akin to the Cassegrain type is described in Smith's *Complete System of Optics* (1738). These "compound reflecting engiscopes"—as such microscopes were called—were not aplanatic, so that although they could give good images at the low numerical aperture of 0·05 – 0·1 (which for many years satisfied astronomers in their corresponding telescope objectives) they gave seriously comatic off-axis images when the N.A. was raised. Further, a large fraction of the N.A. was necessarily obstructed by the shadow of the second mirror : this could in certain circumstances lead to undesirable effects such as a spurious doubling of the number of lines in the image of a grating. Finally, no satisfactory technique had been developed, either for making or testing the required aspheric surfaces. Accordingly, reflecting micro-objectives passed into oblivion. However, in 1905, Schwarzschild gave the analytical solution of the problem of designing an aplanatic (i.e. spherically corrected and coma-free) two-mirror telescope objective. Algebraically this is of course also the solution of the problem of the two-mirror micro-objective when used at infinite or very great tube length, though the range of numerical values of Schwarzschild's two design parameters m, ϵ, corresponding to practical designs of micro-objective, differs from that corresponding to practical designs of telescope objective. Schwarzschild's classic memoir seems to have attracted relatively little attention, for in 1922 the problem of the two-mirror aplanat was attacked independently by Chrétien, at the instigation of G. W. Ritchey, who had been struck by the fact that the 60″ telescope at Mount Wilson was more nearly aplanatic when used as a Cassegrain than as a Newtonian. "He suspected," writes Chrétien, "that the introduction of the hyperbolic mirror produced a kind of compensation of the aberration of the parabolic mirror : he asked me to study this system theoretically, and in particular, to enquire if it was not possible to improve the imaging properties further by freely forsaking the parabolic and hyperbolic shapes previously given to mirrors."

One of the first to take up afresh the study of reflecting microscopes seems to have been D. D. Maksutov, who, in U.S.S.R. patent No. 40859 (1932), mentions the use of the sphere-cardioid aplanatic pair as a micro-objective,[1] and shows an extremely ingenious "solid" reflecting objective in which the same air-glass surface, after acting as "first mirror" by total internal reflection, lets out the image formed by the second (silvered) surface at normal incidence. His work does not appear to have been suggested by that of Schwarzschild or Chrétien, whose general formulae are not quoted.

More recently he has designed reflecting objectives composed of two spherical mirrors with one or more relatively weak lenses on the image side. Photomicrographs taken in U.V. with one of these objectives have been published by E. M. Brumberg and others. The combination of reflection and refraction has been exploited in a different way by B. K. Johnson, who has published U.V. photomicrographs taken with a reflecting objective analogous to the Mangin mirror, while E. H. Linfoot has combined more reflection with less refraction in two reflecting objectives of Schmidt type, which he showed at the Physical Society's exhibition in 1939.

§ 2. SCHWARZSCHILD APLANATS

Consider the reflecting objective shown in figure 1, consisting of a concave mirror of paraxial radius ρ and a convex of radius R separated by a distance ϵ, the unit of length being the focal length of the combination. Parallel light is imaged at distance m from the pole of the ρ-mirror. ρ, R, ϵ, m are all positive in the diagram. Schwarzschild showed that the shapes of the two mirrors may be so defined that the system is spherically corrected and satisfies the sine condition, and he solved the resulting differential equations in finite form, obtaining for the ρ-mirror the equation

$$\frac{1}{r}=\frac{t}{\epsilon}+\frac{1}{m}\frac{\left(1-\frac{t}{\epsilon}\right)^{\frac{1}{1-\epsilon}}}{(1-t)^{\frac{\epsilon}{1-\epsilon}}} \qquad \ldots\ldots(1)$$

where

$$t=\sin^2 \tfrac{1}{2}u \qquad \ldots\ldots(2)$$

and r is the length of a ray leaving the object-point at angle u to its incidence-point on the ρ-mirror.

The incidence point of this ray on the second mirror may conveniently be expressed in Cartesian coordinates:

$$\left.\begin{aligned} x&=\epsilon-(1-t)\left[\frac{r(\epsilon-2t)+t}{\epsilon-rt}\right]\ldots\ldots \\ y&=2t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}=\sin u\ldots\ldots \end{aligned}\right\} \qquad \ldots\ldots(3)$$

The incidence angle i' of this ray on the first (ρ) mirror is given by

$$\tan i'=\frac{(\epsilon-r+1-t)}{\epsilon-t}\left(\frac{t}{1-t}\right)^{\frac{1}{2}}. \qquad \ldots\ldots(4)$$

Equations (1) to (4) define the aplanat in terms of m and ϵ, so that we have to choose numerical values for these parameters.

I do not propose to discuss the design-problem exhaustively, but shall give, for the convenience of those wishing to work on these objectives, the principal approximate formulae with which the designer will find himself concerned. We may be influenced by the coefficient of astigmatism or that of field curvature or by the

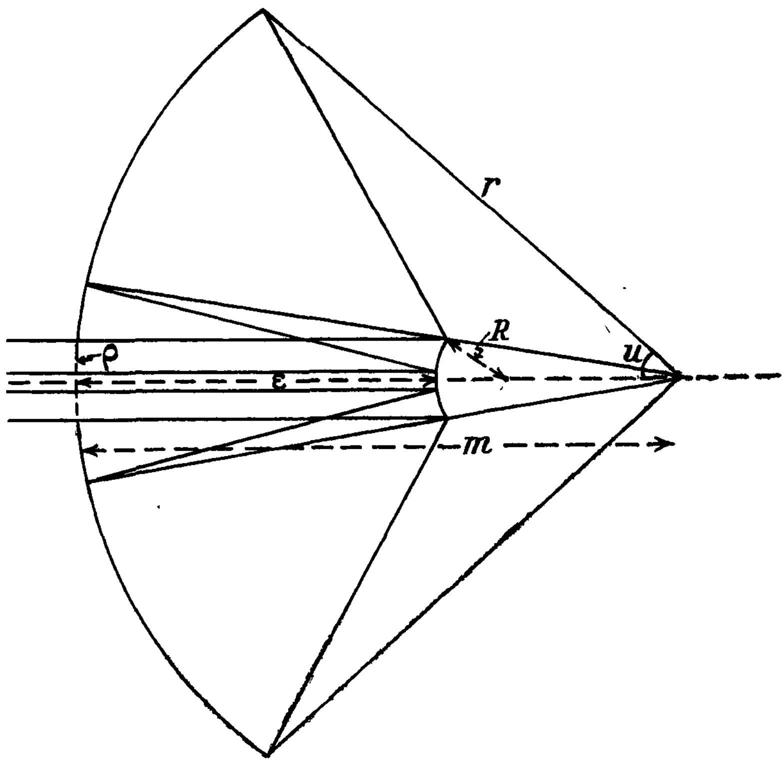

Figure 1. Schwarzschild aplanat.

obstruction ratio or by manufacturing considerations. Schwarzschild showed that the astigmatic interfocal distance which would be needed in object-space to give parallel rays at (small) angle θ to the axis in image-space is

$$\text{A.I.D.} = \frac{2-\epsilon}{2m} \cdot \theta^2, \qquad \ldots\ldots(5)$$

that is, $\frac{2-\epsilon}{2m}$ times the "thin lens" value. He showed also that the surface midway between the focal lines will be curved

$$\frac{(m-1)^2 + \epsilon(1-\epsilon)}{2m\epsilon} \cdot \theta^2 \qquad \ldots\ldots(6)$$

away from the Gaussian image plane.

The fraction of the N.A. shadowed out by the R-mirror is approximately

$$\frac{1}{m-\epsilon} \quad \text{or} \quad m-\epsilon, \qquad \ldots\ldots(7)$$

whichever is less than unity. (In the latter case the light must first pass through a hole in the R-mirror). This approximation contains no unused margin allowance, and is a little over optimistic when the N.A. is not small.

For ease of manufacture we would prefer one mirror to be spherical if this is permissible. Accordingly we seek series approximations to equations (1) and (3). Schwarzschild gives for the ρ-mirror the Cartesian series

$$x=m+\left[\frac{1-m}{\epsilon}-1\right]\frac{y^2}{4m}-\left\{\frac{(1-m)^2}{\epsilon}-\frac{(1-m)}{\epsilon}+\frac{1}{2\epsilon}\right\}\frac{y^4}{16m^3}$$
$$+\left[2\left(\frac{1-m}{\epsilon}\right)^3-2\left(\frac{1-m}{\epsilon}\right)^2+\frac{2}{\epsilon}\left(\frac{1-m}{\epsilon}\right)-\frac{1+\epsilon}{6\epsilon^2}\right]\frac{y^6}{64m^5}\text{, etc.} \quad \ldots\ldots(8)$$

If we disregard terms above y^6, the "non-elliptic" part of this is

$$P_{NE}(x)=\frac{-m}{128\epsilon^2}\left[\frac{1+\epsilon}{3}+\frac{\epsilon}{1-m-\epsilon}\right]\left(\frac{y}{m}\right)^6, \quad \ldots\ldots(9)$$

while the remainder is an ellipse of paraxial radius

$$\rho=\frac{2m\epsilon}{m+\epsilon-1} \quad \ldots\ldots(10)$$

and of eccentricity e' given by

$$e'^2=\left(\frac{1-m+\epsilon}{1-m-\epsilon}\right)^2+\frac{2\epsilon^2}{(1-m-\epsilon)^3}, \quad =e_0'^2-\frac{1}{4\epsilon}\left(\frac{\rho}{m}\right)^3, \quad \ldots\ldots(11)$$

where e_0' is the eccentricity of that ellipse whose foci are the object-point and paraxial intermediate image-point.

For the R-mirror Schwarzschild gives $x=A+By^2+Cy^4+Dy^6+Ey^8$ where

$$\left.\begin{aligned}&A=m-\epsilon\ ;\quad B=\frac{1-m}{4\epsilon}\ ;\quad C=-\frac{1}{8}\cdot\frac{m}{4\epsilon}\ ;\\ &D=-\frac{1}{96}\cdot\frac{1+4\epsilon}{\epsilon}\cdot\frac{m}{4\epsilon};\quad E=-\frac{1}{1536}\cdot\frac{2+11\epsilon+30\epsilon^2}{\epsilon^2}\cdot\frac{m}{4\epsilon}.\end{aligned}\right\} \quad \ldots\ldots(12)$$

This is approximately an ellipse of paraxial radius

$$R=\frac{2\epsilon}{m-1} \quad \ldots\ldots(13)$$

and of eccentricity e given by

$$e^2=1+\frac{2\epsilon^2 m}{(1-m)^3} \quad \ldots\ldots(14)$$

The non-elliptic part of the sixth power coefficient is

$$P_{NE}(D)=D-\frac{2C^2}{B}. \quad \ldots\ldots(15)$$

Let us summarize the main design-restrictions which these formulae imply. For greatest ease of manufacture we equate e' and e simultaneously to zero. This gives $\epsilon=2$, so that the system is anastigmatic, and $m=2+\sqrt{5}$, $\rho=1+\sqrt{5}$; $R=\sqrt{5}-1$, a monocentric objective. The price that we must pay for having zero first coefficient of asphericity on both mirrors—so that they may be spherical up to N.A. approaching 0·5—is the high intrinsic obstruction ratio, $1/\sqrt{5}$. If we wish to retain anastigmatism, we must set $\epsilon=2$, and, except in the case just considered, e' and e both differ from zero for every value of m, so that both mirrors must be aspherized. If we are prepared to give up exact anastigmatism, and be content with aplanatism only, we may set e zero, giving $\epsilon^2=\frac{(m-1)^3}{2m}$, and select m so as to give as small an obstruction ratio, $\frac{1}{m-\epsilon}$, as we please. This gives an aplanat in which—provided terms involving the sixth power of the N.A. may be neglected—the R-mirror may be spherical. Alternatively we can set e' zero, but this leads to a higher astigmatic coefficient for a given obstruction ratio. I show in a separate paper that when the design is optimized, the R-mirror may be spherical in an objective of small obstruction ratio, up to 0·65 N.A. If we ask—may the R-mirror ever be exactly spherical, up to N.A. unity, the answer is yes, provided $\epsilon=\frac{1}{2}$, $m=2$. The Schwarzschild concave curve then becomes a cardioid, and the combination is the well-known sphere-cardioid pair used in aplanatic dark-ground condensers, usually in approximation as two spheres.

§ 3. EXPERIMENTAL

I have made two "through-type" reflecting objectives, which, though they are corrected for finite tube-lengths, may be regarded as approximating to Schwarzschild aplanats having respectively $\rho=4{\cdot}31$ cm., $R=0{\cdot}85$ cm., $m=8$, $\epsilon=4$, $f=0{\cdot}75$ cm.; N.A. ·58, tube-length 32 cm., magnification ×47, and $\rho=5$ cm., $R=0{\cdot}397$ cm., $m=20$, $\epsilon=13$, $f=0{\cdot}3$ cm.; N.A. ·65, tube-length 30 cm., magnification × 100. To this objective I have added a normal-incidence oil-immersion lens, the surface of which is spherical and concentric with the axial object point. This raises the N.A. to 0·98, and the magnification to about 152, the magnification now being proportional to the refractive index of the lens.

The first objective has both mirrors aspherized: the concave mirror was first aspherized so as to annul spherical aberration for a certain arbitrarily chosen separation between the mirrors and object position, after which the convex mirror was moved axially with respect to the concave so as to annul off-axis coma (the final image position being unchanged). The spherical aberration re-introduced by this change was then annulled by aspherizing the convex mirror. This procedure produces a sufficient approximation to aplanatism: a more elaborate procedure would be needed for N.A.$>$0·7.

Local figuring was needed to restore revolution symmetry lost during aspherizing, and the residual error is of "cobbled pavement" type, of the order of $\frac{1}{8}$ fringe by double transmission. The leading off-axis error for image points 1 cm. off-axis is a fraction of a fringe of astigmatism.

The second objective has a nominally spherical convex mirror, the spacing between the mirrors being adjusted to give the best compromise between high-order and Seidel coma; the leading error for image points $\frac{1}{2}$ cm. off-axis consists of a fraction of a fringe of compromise—comatic retardation of the edge of the wave-front remote from the axis: for image points 1 cm. off-axis the leading error is astigmatism. This objective also has a small fraction of a fringe of "cobbled pavement" error.

I find it advisable, with both objectives, to stop out, in the substage condenser, that part of the N.A. which is obstructed in the objective. If this is not done, the contrast is poor when the substage N.A. is reduced, as is to be expected, for if the substage N.A. is reduced until only the cone obstructed by the objective is supplied, the conditions are those of "patch-stop dark-ground illumination" and black objects are seen bright on a black ground. [If a dark-ground effect is desired, I find it better to use a standard dark-ground illuminator supplying a cone lying outside that used by the objective, as the diffraction rings surrounding images are then much less prominent.]

The visual performance of both objectives used with full substage aperture on stained specimens seems comparable in contrast and resolving power with that of refracting objectives of equal N.A.

Some idea of the photographic performance may be obtained from figures 2–6, which are reproduced from enlargements made by Mr. H. Busby of films taken by Dr. G. P. Occhialini with the second objective. For their kindly interest and enthusiastic co-operation I record my grateful thanks.

Figures 2 and 3, taken without the oil immersion component, show "thorium stars" ×1140, and treponema pallida, silver-stained by Levaditi's method, × 950, 0·65 N.A. ; 0·65 substage N.A.

Figures 4, 5 and 6 were taken with oil immersion : figure 4 shows part of the long chromosome of Drosophila, ×1420, ·98 N.A., ·45 substage N.A. ; figure 5 shows treponema pallida, Levaditi stained, ×2230, and figure 6 gonococci in pus cells, stained with methylene blue, ×2230, 0·98 N.A., 0·98 substage N.A. The light source in all cases was a half-watt projector lamp, diffused by ground glass, with a green filter. No eyepiece was used, the film being placed at the primary image.

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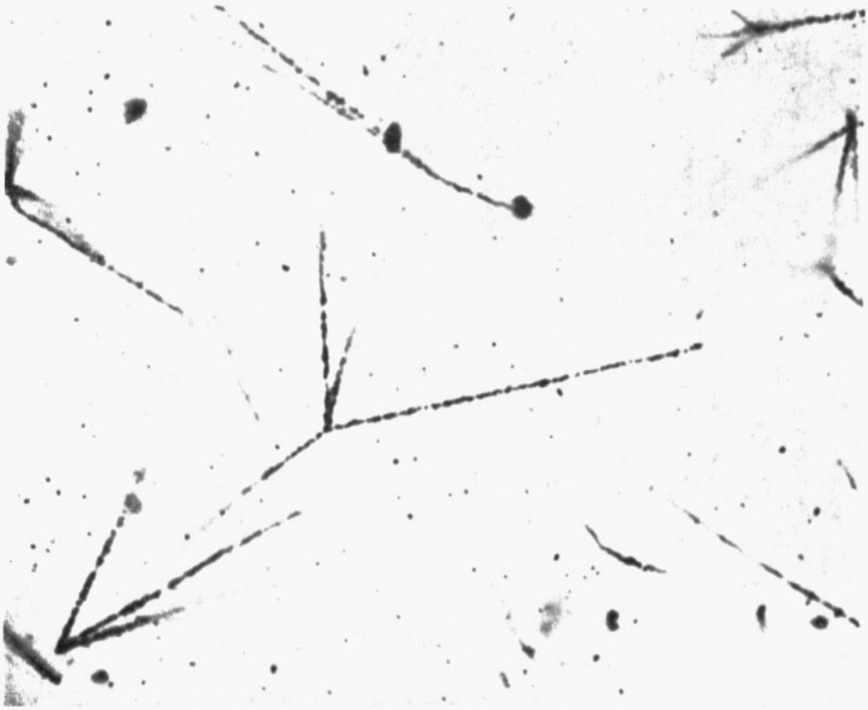

Figure 2.

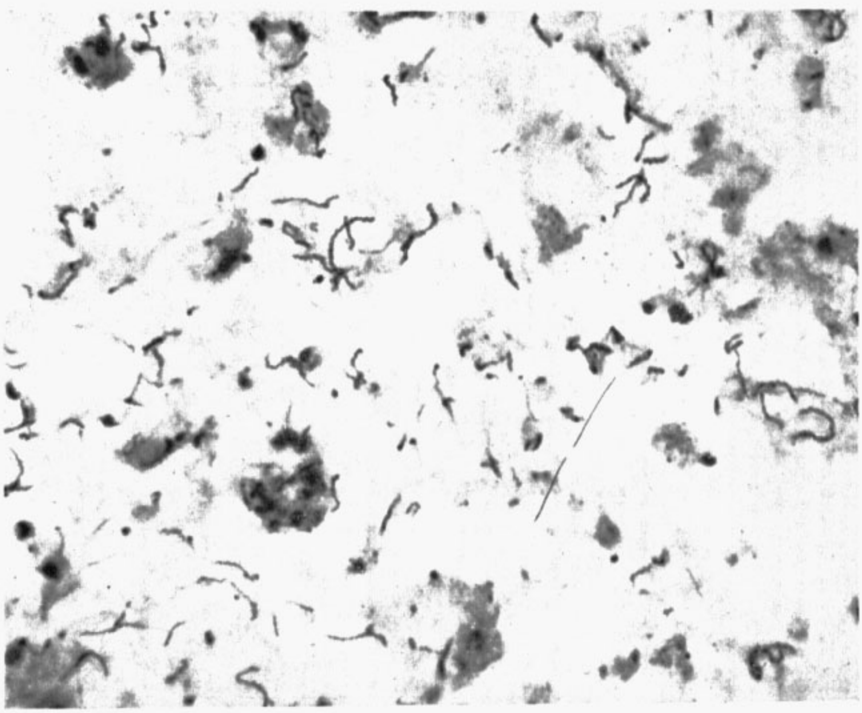

Figure 3.

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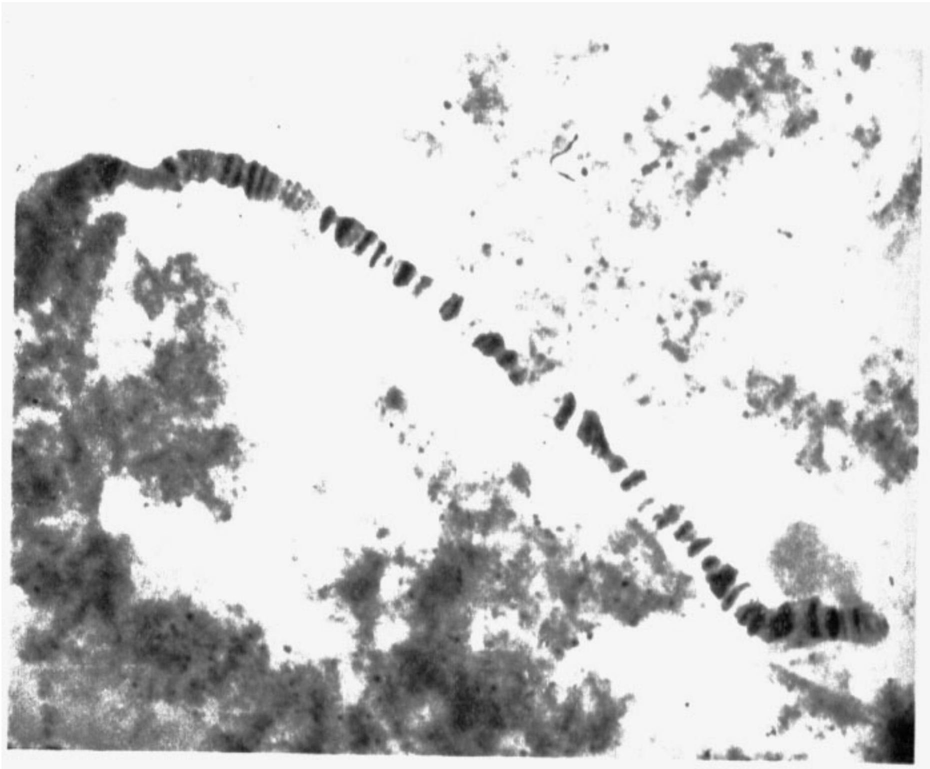

Figure 4

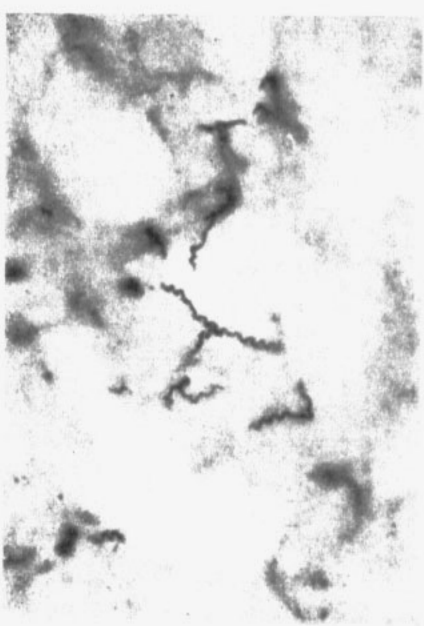

Figure 5.

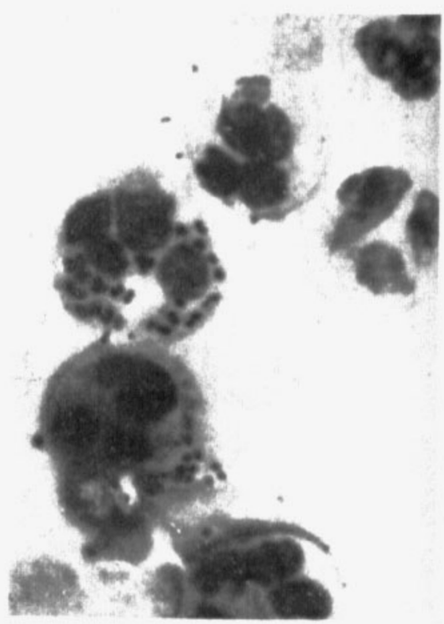

Figure 6.